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PROBABILISTIC OBSERVATIONS ON ANTISUBMARINE
WARFARE TACTICAL DECISION AID (ASWTDA)

by

Bruce R. Bjorklund

March, 1990

Thesis Advisor:

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Probabilistic Observations On
Antisubmarine Warfare Tactical
Decision Aid (ASWTDA)

by

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Lieutenant, United States Navy
B.S., Iowa State University, 1983

Submitted in partial fulfillment
of the requirements for the degree of

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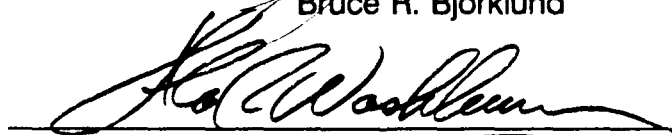
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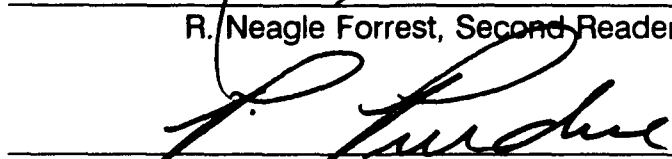
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ABSTRACT

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First, a Classical Computer Assisted Search program is described as a basis of comparison for the methodology employed in ASWTDA. Then, the operations as performed in ASWTDA are described, followed by a probabilistic analysis. In the analysis sections, probabilistic support for the applied methodology is provided where applicable, and conceptual problems and possible solutions are cited where appropriate.

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I. INTRODUCTION

A. THESIS GOAL

The goal of this thesis is to examine the methodology used in ASWTDA from a probabilistic standpoint. It is intended to provide probabilistic support for the applied methodology where applicable and to cite conceptual problems and suggest possible solutions where appropriate.

B. ASWTDA GOALS

ASWTDA is a tool to assist platform, unit or force commanders afloat and ashore in making tactical ASW decisions. It is necessary that it support rapid assimilation of data pertinent to the problem and needed in the decision making process. ASWTDA is to be an aid in the Search and Detection phases of the ASW prosecution effort during which the target's position is not known with sufficient accuracy for weapon placement [Ref. 1: p. v]. Target localization and weapon control is to be performed by existing, onboard fire control systems. ASWTDA is a decision support tool to assist in the conduct of the following [Ref. 2: p. 1]:

1. Large area environmental analysis
2. Large area search planning

3. Resource allocation and management
4. ASW contact correlation and management

C. OVERVIEW OF ASWTDA OPERATION

ASWTDA begins operation at some problem start time with an initial target location distribution and an assumed target motion model. ASWTDA then processes data from negative search efforts, positive contact reports and target motion to produce, at some future forecast time, a probability distribution for the target's position. This data may then be displayed as a probability map showing regions where target location is more or less likely, or it may be combined with environmental and sensor data to produce a map showing regions where target detection is more or less likely.

D. ASWTDA BACKGROUND

The development of ASWTDA began about November 1988. Chief of Naval Operations OP-71, AntiSubmarine Warfare Division, was named sponsor of the program, NAVSEA 63-D3 was named program manager. OP-71 and NAVSEA then named Naval Oceanographic Systems Center (NOSC) of San Diego, CA as Technical Director, Naval Underwater Systems Center (NUSC), New London, CT as developer and Naval Air Development Center (NADC), Warminster, PA as the laboratory for independent verification and validation. [Ref. 3]

It was felt that existing computer search aids were not adequate for the ASW commanders and operators at sea. OP-71 established a fleet working group to provide requirements for a new ASWTDA to meet the Navy's needs. A major goal was to get the program to the fleet as quickly as possible. The development was to be an evolving process with rapid prototyping and fleet feedback. In order to reduce development effort, the program was to evolve from three previous efforts. These were [Ref. 3]:

1. Integrated Tactical Decision Aid (ITDA)
2. Composite Area Search Evaluation (CASE)
3. Fast Frigate Integrated Sonar Tactical System (FFISTS).

The analysis performed in this thesis is based on the methodology as described in Reference 1. This was the methodology in use as of November 1989. Due to the nature of the development process of ASWTDA, the program may change greatly in a short period of time. The reader must bear in mind that this is based upon dated information and the program as analyzed is not necessarily what is being, or has been delivered to the Navy for fleet use. As of this writing, a version of ASWTDA is in place for at sea testing onboard the U.S.S. Cushing, the flagship of Destroyer Squadron 31, an ASW squadron based in San Diego, California.

E. ORGANIZATION

Chapter II describes a model of a classical Computer Aided Search (CAS) program. This will provide the reader with a general background of the operations performed in such a program and the notation used in the description of the operations of ASWTDA.

In Chapters IV and V, the following specific areas will be addressed in this analysis:

1. Data Fusion
2. Target Motion

Each chapter is broken down into two major sections. In the first section, **Methodology**, a description is given of the mechanics of the operations involved, with no attempt to discuss the reasoning behind the operations. In the second section, **Probabilistic Analysis** an attempt is made to provide probabilistic justification for the methods employed.

II. COMPUTER ASSISTED SEARCH, A CLASSICAL MODEL

A Classical model of a moving target CAS program is a program which outputs a probability map of target position at any time specified by the operator. The three main elements of a Markovian moving target CAS system are:

1. Prior Probability map of target Position
2. Model of Target Motion
3. Updates of the probability map for target motion and information received on the target

Advanced programs of this type may also provide the user with search plan recommendations.

A. PRIOR PROBABILITY MAP

In CAS applications, the geographic region of interest is always divided into an array of cells. The target location is described by a probability distribution over these cells. We will index the cells in the array with the subscripts i,j : $i=1...N$, $j=1...M$; this set of cells makes up the analysis area, S . Let $E_{i,j}(t)$ be the event a target is in cell (i,j) at time (t) and define $p_{i,j}(0)=P(E_{i,j}(0))$, ie. $p_{i,j}(0)$ is the probability the target is located in cell (i,j) at time (0) .

The prior probability map is constructed by assigning a number between 0 and 1 to each p_{ij} at time $t=0$ such that:

$$\sum_{i=1}^N \sum_{j=1}^M p_{ij}(0) = 1.0 \quad (1)$$

This distribution is called the "prior probability map of the target", frequently referred to simply as the "prior". If the operator has no information about target location, the prior may be simply a uniform target distribution.

B. TARGET MOTION MODEL

Target motion must be described in probabilistic terms because of the uncertainties involved with the motion. A common motion model that is used is the Markov-chain model. The target location is known as the *state* of the target. Some proportion of the probability moves from each cell to the others in each time step, as determined by a transition matrix, Q . The Markov-chain model contains no target history other than what is contained in the state. Thus the cell probabilities change at a given time based only on the information available in the cell at that time (the definition of a Markov process) [Ref. 4: p. 21].

C. PROBABILITY MAP UPDATING

1. Search Information

Search information may be of two types: 1. Negative information: search is performed, but no target was found. 2. Positive information: a report on target

position resulted from search. For our purposes, local sensors (sonobuoys, towed arrays, etc.) provide only negative information. If target detection occurs by these sensors, the problem is one of localization and is no longer a wide area search problem of the type addressed by ASWTDA. Positive information may be obtained from a long range type sensor, e.g. SOSUS, which does not provide a target position with sufficient accuracy for target localization without further search.

Updating for search information is done by application of Bayes' theorem. Let $I(t)$ be the event that some information has been received about the target at time (t) . Assuming that $P(I(t) | E_{ij}(t))$, i.e., the probability that information $I(t)$ is received, given the target is in cell (i,j) at time (t) , is known for all i,j , the target's position distribution can be modified on the basis of receipt of this information. These probabilities are based on models of search effectiveness for all types of search performed in the analysis area. The target distribution is modified as follows [Ref. 5: p. 65]:

$$P(E_{ij}(t) | I(t)) = \frac{P(I(t) | E_{ij}(t))P(E_{ij}(t))}{\sum_{i,j \in S} P(I(t) | E_{ij}(t))P(E_{ij}(t))} \quad (2)$$

Note that $P(E_{ij}(t) | I(t)) = P(E_{ij}(t))$ when $P(I(t) | E_{ij}(t))$ is independent of (i,j) . This is commonly assumed to be the case when $I(t)$ is "null", i.e., no information is received.

For the purposes of this paper, information is received about the target at discrete times Δt apart. These times will be denoted by $t, t+1, t+2, \dots$. Let:

$$\begin{aligned} p_{ij}(t) &= P(E_{ij}(t) | I(1), \dots, I(t-1)) \\ p_{ij}^*(t) &= P(E_{ij}(t) | I(1), \dots, I(t)) \end{aligned} \quad (3)$$

Equation (2) becomes:

$$p_{ij}^*(t) = \frac{P(I(t) | E_{ij}(t)) p_{ij}(t)}{\sum_{i,j \in S} P(I(t) | E_{ij}(t)) p_{ij}(t)} \quad (4)$$

This requires an assumption about independence, however. Note that the $p_{ij}(t)$ are actually conditional probabilities, with the condition being the information history $H_t = (I(0), \dots, I(t-1))$, prior to receipt of $I(t)$. Thus $p_{ij}(t) = P(E_{ij}(t) | H_t)$, and the problem is to calculate $P(E_{ij}(t) | I(t)H_t)$. Bayes' theorem is actually:

$$P(E_{ij}(t) | H_t I(t)) = \frac{P(I(t) | H_t E_{ij}(t)) P(E_{ij}(t) | H_t) P(H_t)}{\sum_{i,j \in S} P(I(t) | H_t E_{ij}(t)) P(E_{ij}(t) | H_t) P(H_t)} \quad (5)$$

where $P(H_t E_{ij}(t)) = P(E_{ij}(t) | H_t) P(H_t)$. We can then cancel $P(H_t)$ and Equation (5) becomes:

$$P(E_{ij}(t) | H_t I(t)) = \frac{P(I(t) | H_t E_{ij}(t)) p_{ij}(t)}{\sum_{i,j \in S} P(I(t) | H_t E_{ij}(t)) p_{ij}(t)} \quad (6)$$

which is equivalent to Equation (2) if it is true that $P(I(t) | H_t E_{ij}(t)) = P(I(t) | E_{ij}(t))$; i.e., conditional on target location being given, $I(t)$ is independent of H_t . This assumption is implicit in the repeated application of Bayes' theorem, i.e., each new piece of information must be conditionally independent of past information. This is usually a reasonable assumption, with the exception of systems such as SOSUS, which

gives frequent reports that are correlated, even when target position is given.
[Ref. 6: p. 7]

2. Target Motion

In the Markov-chain model, the motion update requires an update of the target probabilities in each cell. Given that a target is in cell (i,j) , it may, in a given time interval, remain in cell (i,j) , or transition to another cell in the analysis area. Associated with each cell (i,j) , is a transition matrix, Q , which describes target motion from cell (i,j) in one time step. Let $Q(i,j | m,n)$ be $P(E_{ij}(t+1) | E_{m,n}(t))$. Then, assuming Markov target motion [Ref. 4: p. 240]:

$$p_{ij}(t+1) = \sum_{n \in S} \sum_{m \in S} p_{m,n}^+(t) Q(i,j | m,n) \quad (7)$$

Thus, as the notation is used here, $p_{ij}(t+1)$ is obtained from $p_{ij}^+(t)$ by application of the transition matrix, Q , for target motion during the time interval (t) to $(t+1)$. $p_{ij}^+(t)$ is obtained from $p_{ij}(t)$ by application of Bayes' theorem for information received at time (t) . The sequence of computations thus goes: $p(0)$, $p^+(0)$, $p(1)$, $p^+(1)$, $p(2)$, etc.

In principle, the target state may be much more complex than simply target location. It may contain other properties such as target course, depth, motion model, time, etc. The transition matrix, Q , would then contain transition probabilities for each of these states to every other possible state. This more general notation will not be used in this thesis however, because the state in ASWTDA consists only of position.

III. TERMINOLOGY AND NOTATION

A. TERMINOLOGY

ASWTDA performs analysis within a geographic region with location and size specified by the operator at the beginning of the session. The region is a square ranging from 100 to 500 Nautical Miles on a side. This region, known as the Analysis Area, is then fixed and may not be changed without problem re-initialization. This analysis area is subdivided into a grid of 1764 (42 by 42) square cells. Thus, cell size dimensions range from 2.38 to 11.90 nautical miles. Target location is maintained in ASWTDA as a probability distribution over the cells.

ASWTDA maintains two target density matrices:

1. Area Clearance Density Matrix
2. Primary Target Density Matrix

The purpose of the Area Clearance target density display is to illustrate target density in the absence of target detection. It is a display to be used primarily for determining regions which have been searched and sanitized, thereby containing a low probability of a submarine threat. These are areas into which the operational commander may safely send vulnerable forces.

The Primary Target density display shows the Primary Target density at any time after target detection has occurred. This is intended to be the display to which

the operator focuses his attention following target detection, although the Area Clearance display may be monitored for the reasons stated above.

The operator may also view a combined density display which is formed from the sum of the Area Clearance and Primary Target density matrices and is calculated at the time of display.

B. NOTATION

Each density matrix is described as a 42 by 42 cell array with AC_{ij} representing the probability density for a particular cell in the Area Clearance matrix, and PT_{ij} the probability density for a cell of the Primary Target density matrix.

For a matrix X , the norm, $|X|$, is defined by:

$$|X| \doteq \sum_{(i,j) \in S} X_{i,j} \quad (8)$$

Also, the product $X \times Y$ for matrices X and Y is a multiplication on a cell by cell basis of the values of X_{ij} and Y_{ij} , rather than the matrix product of Linear Algebra.

The notation as described in Chapter II with regard to information and position updating will be utilized for describing the operations through time on the target density matrices.

For any two events x and y , when these are listed together in a probability statement, the meaning is (x and y), e.g., $P(xy) = P(x \text{ and } y)$.

IV. DATA FUSION

A. METHODOLOGY

The target density matrices are updated for information at 6-minute intervals. This update occurs at the end of each 6-minute period. At each update, the negative effects of all search efforts are incorporated, then diffusion due to target motion is applied.

ASWTDA incorporates negative information from three types of acoustic search: sonobuoys (single sonobuoys and sonobuoy fields), search tracks and search areas. A search track is used when the sensor platform's track is known with reasonable accuracy. A search area is used when a bounded region is assigned to a search platform, but the platform's motion within the region is not specified. [Ref. 1: p. 53]

1. Updating Target Density for Information

a. Prior to Receipt of Positive Information ($t < t_0$)

On problem initialization, ASWTDA assumes a single target is distributed uniformly over the analysis area, thus $p_{ij}(0) = 1/N$ for all non-land cells, where N is the number of non-land cells within the analysis area, S . Prior to receipt of positive information on a target, this probability distribution is contained entirely within the Area Clearance target density matrix, thus $AC_{ij}(0) = p_{ij}(0)$ for all $(i,j) \in S$, and the Primary Target density matrix is initialized to 0, so, $PT_{ij}(0) = 0$ for all $(i,j) \in S$.

Let $I(t)$ be the event that no target detection occurs between times $(t-1)$ and (t) . For a position update at time (t) , the cumulative probability of detection for each cell over the six-minute time interval from $(t-1)$ to (t) is calculated based on all search efforts in progress during the time interval from $(t-1)$ to (t) . These calculations are based on search models for each type of search and incorporate environmental data as well as data about the sensors themselves. Let $Pn_{ij}(t) = P(I(t) | E_{ij}(t))$, i.e., $Pn_{ij}(t)$ is the probability that no detection occurs in cell (i,j) during the time interval $(t-1)$ to (t) , given the target is in cell (i,j) . The Area Clearance target density matrix is then updated by applying Bayes theorem to all cells as follows:

$$AC^+(t) = \frac{AC(t) \times Pn(t)}{|AC(t) \times Pn(t)|} \quad (9)$$

b. Upon Initial Receipt of Positive Information ($t=t_0$)

ASWTDA assumes that the target distribution based on the positive information can be described by a bivariate normal distribution. Let B represent this bivariate normal distribution. This distribution is discretized into the same size cells as used in the ASWTDA analysis area and probability values are assigned to the cells according to the amount of the bivariate normal density encompassed by the cell. The discretized bivariate normal density will be denoted B^D . Note that B^D extends infinitely in all directions, whereas S covers only a finite area.

This target distribution is assigned a confidence factor (cf), currently set at a value of 0.8 but designed to be an operator assigned value in future program

releases. The discretized bivariate normal distribution is renormalized so that $|B^D| = cf$. The confidence factor is intended to be representative of the input distribution as a whole and not a function of any individual contact reports used to generate the input distribution. [Ref. 1: p. 74]

As a result of this positive information, $PT_{ij}(t_0) = B^D_{ij}$ for all $(i,j) \in S$, provided the cell does not fall on land, in which case it is assigned a value of zero.

The distribution contained in

$PT(t_0)$ then is that subset of B^D

which is contained in S as

illustrated in Figure 1. Let

$pr = |PT(t_0)|$. pr will be

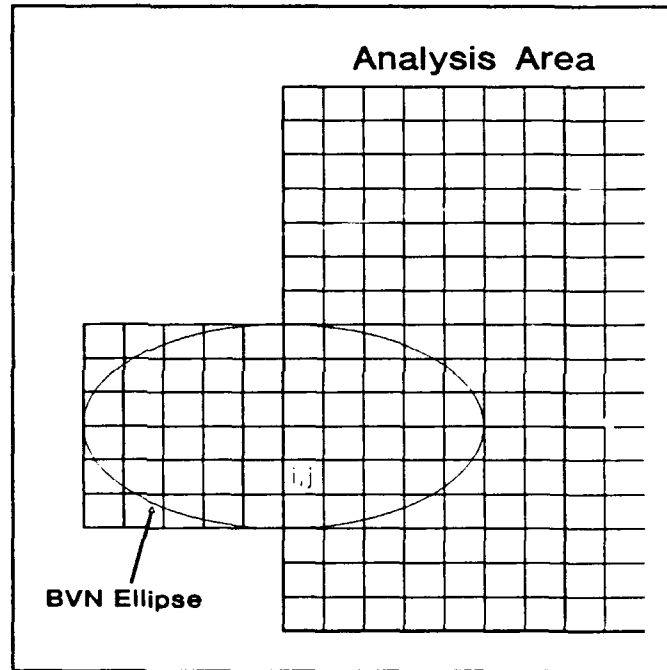
dependent upon the amount of

B^D which falls within S , not on a

land mass. In all cases, $pr \leq cf$;

for example, for the case

illustrated in Figure 1, $pr \approx cf/2$.



The following **Figure 1** Bivariate Normal Distribution Overlaid on Analysis Area, S .

operation is then performed to

fuse the previous negative information with this positive information:

$$PT^*(t_0) = \frac{pr \times (AC^*(t_0) \times PT(t_0))}{|AC^*(t_0) \times PT(t_0)|} \quad (10)$$

2. Subsequent to Receipt of Positive Information ($t > t_0$)

For each six-minute time interval, negative search information is incorporated into the Area Clearance density matrix using Equation (9) and the Primary Target density matrix using Bayesian updating in the same manner as follows:

$$PT^*(t) = \frac{PT(t) \times Pn(t)}{|PT(t) \times Pn(t)|} \quad (11)$$

3. Updating Target Density for Further Positive Information

Currently, additional positive information may be treated in one of two ways:

1. The new report may be incorporated into the Over the Horizon-Maneuvering Target Statistical Tracker (OTH-MTST) input data set to produce a new initial target distribution.
2. The contact report may be used as the goal of a constrained random walk scenario, with a past report as the initial distribution for the scenario. In this manner, several reports may be incorporated in a series of constrained random walks.

B. PROBABILISTIC ANALYSIS

As described above, negative information is incorporated into both target density matrices according to Bayes' theorem. It can be shown that the initial positive information is also incorporated in a Bayesian manner if certain assumptions are made about the received information, and the target is assumed to have followed the motion model of the Area Clearance target prior to the information update.

Specifically, if at time (t_0), it is true that the distribution of the Primary Target is equivalent to the distribution of the Area Clearance target, and $pr=1$, then Equation (10) is a proper application of Bayes' theorem.

There are two issues that arise, however:

1. Why is the distribution of the Primary target equivalent to the distribution of the Area Clearance target prior to the positive information update?
2. Why is $pr \neq 1$?

These issues will be discussed in Chapter VI.

Further positive information, however, is not incorporated in a Bayesian manner. Either of the methods mentioned above which may be utilized to incorporate this further positive information results in a change of the Primary Target motion model.

It is possible that, given more than one possible target motion model, receipt of positive information may alter the probabilities that we assign to each of these models. In other words, we change our belief as to what the Primary Target is, and has been doing, based on this information. This is an important point in that the target motion prior to the receipt of the positive information must also be taken into account. This is not the case in ASWTDA. Based on receipt of positive information at time (t_0), the Primary Target changes its motion model, i.e., at time (t_0), the Primary Target distribution is dependent upon the Area Clearance target motion model prior to time (t_0). For $t > t_0$, the Primary Target then follows an entirely

different motion model. The Primary Target can not be expected to actually alter its motion based solely on the receipt of information on the target, unless it is believed that the target is reacting to our search efforts.

V. TARGET MOTION

A. METHODOLOGY

ASWTDA models target motion using a Markov-Chain approach. Note that in the general notation of Markov motion as described in Section II.C.2, the transition matrix, Q , is a 1764 by 1764 matrix, i.e., it describes the probability of transition from any state to any other possible state. In ASWTDA, however, the target is limited to traveling at most one cell length. This requires a much smaller transition matrix, specifically a 3x3 transition matrix, q_{ij} , for each cell $(i,j) \in S$. This transition matrix describes target motion from that cell during any single motion time interval. A portion of the cell's target density remains in the original cell; the rest is distributed to the adjacent eight cells. [Ref. 1: p. 23]

1. Area Clearance Target diffusion

ASWTDA assumes that a target associated with the Area Clearance Display moves about randomly. The transition matrix, q^{AC} , associated with this motion is shown in Figure 2. This transition matrix is applied to every cell within the analysis area, thus the lack of the i,j subscript. The transition matrix coefficients are fixed, regardless of cell size or target speed.

The transition matrix coefficients were calculated as follows: [Ref. 1: p. 26]

1. Build a 3x3 matrix of 9 square cells.

2. Populate the center cell with 1,000,000 uniformly distributed points.
3. Move each point one cell-length in a random direction (from 0 to 360 degrees).
4. Count the number of points in each of the 9 cells.
5. Normalize the final distribution of points and adjust to ensure symmetry.

At each diffusion iteration,

the target density is re-computed using a temporary density matrix. Let the

temporary matrix be called T. The transition matrix is applied as follows:

1. Zero the temporary matrix, T.
2. For a cell (i,j) of the Area Clearance density matrix, AC, multiply each of the values of the transition matrix, q, by AC_{ij} .
3. Add these values to the matrix T. Thus, the density from cell (i,j) is distributed in cell (i,j) and its eight surrounding cells in matrix T. (See Figure 3)
4. Repeat for every cell (i,j).
5. Replace AC with T.
6. Repeat the process if more transition matrix applications are required.

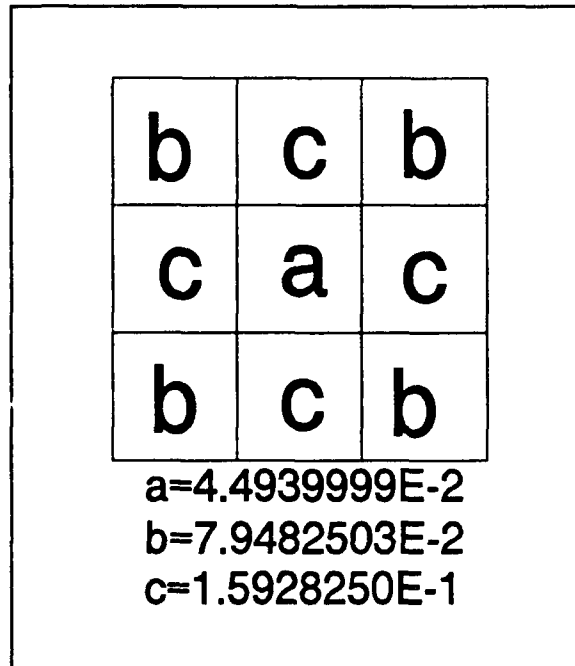


Figure 2 Area clearance Transition Matrix

At edge and corner cells, any density that would diffuse out of the analysis area is retained in the original cell (i,j). At land boundaries, any density that would diffuse onto land is distributed into cell (i,j) and its immediate surrounding non-land cells. Note that analysis area boundary and land effects do not change the transition matrix q^{AC} , but are accounted for at each application of q^{AC} . We will use the following compact notation to represent this:

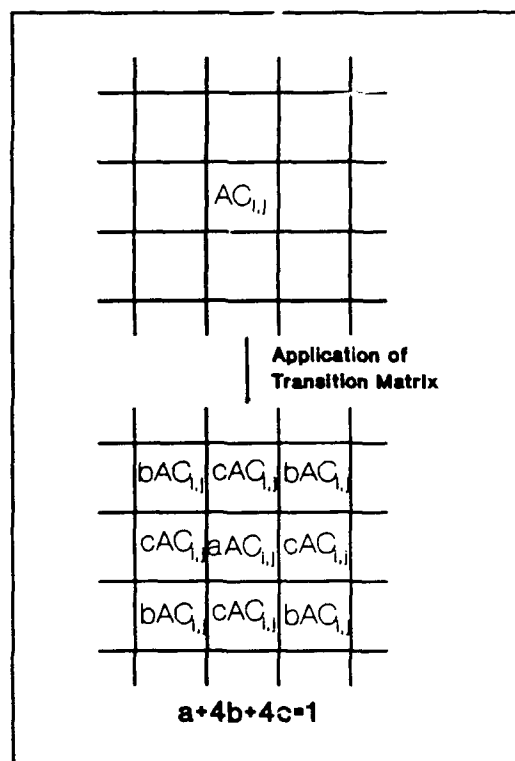


Figure 3 Application of Area Clearance Transition Matrix

$$p(t+1) = p^*(t) \star q \quad (12)$$

Because the coefficients for this matrix are fixed, the Area Clearance target transition matrix is applied using Equation (12), an irregular number of times per six-minute time interval, determined by target speed, mean time on a given course and cell size. In this discussion, an *iteration* is one six-minute interval, while an *application* of the transition matrix refers to a single use of equation (12). This update occurs as follows [Ref. 1: p. 26]:

1. Calculate $R86$, the radius, in nautical miles, which contains 86 percent of the target density. For a given target speed (V) in Kts., mean time on a given course (λ) in hours and length of time (T) in hours, $R86$ is approximated by:

$$R86 = 2\sqrt{V^2\lambda^2\left(e^{-\frac{T}{\lambda}} - 1 + \frac{T}{\lambda}\right)} \min(1.0, 0.707 + 0.293 T/3\lambda) \quad (13)$$

2. Calculate the number of transition matrix applications, n , that would produce an $R86$ equal to the $R86$ approximated by the calculation in step 1, where:

$$n = \frac{(R86/\text{Cell Length})^2}{2.45} \quad (14)$$

3. Since n may not be an integer value, the calculated value is rounded to the nearest whole number. If the required number of transition matrix applications for the current problem time (total time from $t=0$ to time t) is greater than the number of applications performed as of the end of the previous iteration, then the diffusion process is called and the transition matrix is repeatedly applied using Equation (12) until the number of applications performed equals the required number of applications computed in step 2.

2. Primary Target Diffusion

For Primary Target displays, a unique 3x3 transition matrix is calculated for each cell within the analysis area. These transition matrices will be denoted by q^{PT}_{ij} . These matrices are calculated by performing a Monte Carlo simulation using 2000 tracks originating from a specified initial target distribution, and proceeding through an anticipated target motion scenario. Land avoidance is performed in the process of the Monte Carlo simulation thus preventing the Primary Target from diffusing onto land. As a given Monte Carlo track proceeds, the transition matrix building subroutine keeps track of the number of transitions from any one cell to an adjacent cell in the matrix at intervals of dt , given by:

$$dt = \frac{\text{Cell Length}}{(\text{Average Target Speed})(\text{Cos}(\delta))} \quad (15)$$

where δ is the difference from the average target course to the nearest cardinal heading, so $-\pi/4 < \delta < \pi/4$. The cosine term is used because it was determined experimentally that a target on a diagonal course tended to diffuse too quickly in actual application. The simulation process is illustrated in Figure 4. There is a 3x3 matrix associated with each of the cells in the upper matrix. The target track is marked off in numbered intervals of dt . Thus

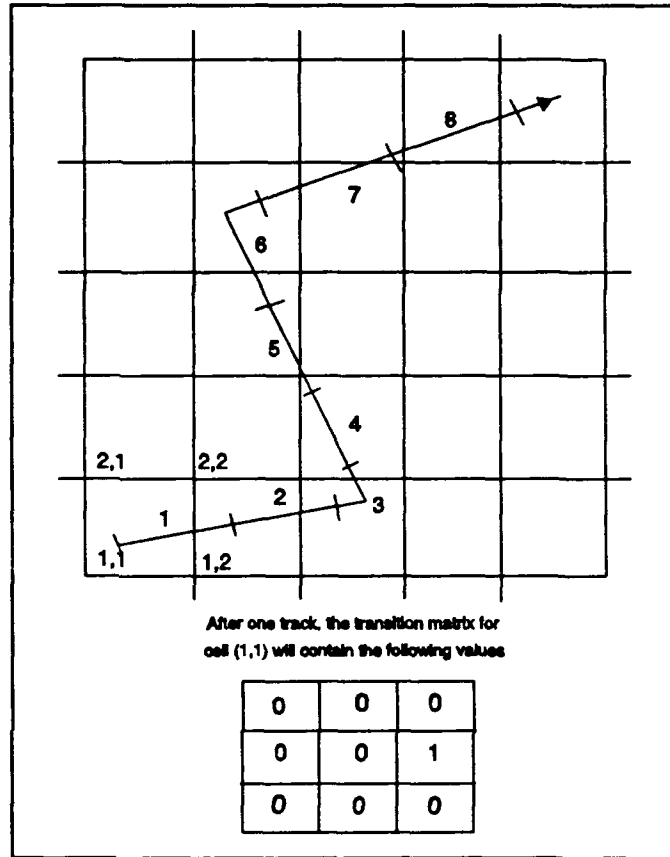


Figure 4 Simulation of One Track for Primary Target Diffusion

after completion of this track, the 3x3 matrix for cell (1,1) would contain the numbers shown in the matrix at the bottom. Because dt is calculated using the average target speed, it is possible that the target may travel farther than one cell length in a time step dt , this is illustrated in time step 8 of Figure 4. If this occurs, a count is placed in the nearest adjacent cell. The simulation then continues from the actual simulated position. After all 2000 tracks have been generated, the cell counts for each 3x3

matrix are translated into transition probabilities. This is performed by replacing each cell of the 3x3 transition matrix with the number of counts in the cell divided by the total number of counts in the 3x3 matrix.

This set of transition matrices is used until additional positive information is received or the target motion scenario is modified, at which time a new Monte-Carlo simulation is performed and the transition matrices are re-computed. The Primary Target transition matrices, q^{PT}_{ij} , are applied using Equation (12) at regular time intervals equal to dt as calculated in Equation (15).

B. PROBABILISTIC ANALYSIS

1. Area Clearance Target Diffusion

The model that ASWTDA assumes for the Area Clearance target is that of a Random Tour. Under this model, the target is assumed to move at constant speed, but to make course changes at random times. Each new course is chosen from a uniform distribution on $[0, 2\pi]$. The random travel time between course changes is assumed to be drawn from an exponential distribution. Each new course and time on course are assumed independent. Note that this is not the same as a diffusion process, which is not a constant velocity process.

ASWTDA uses the results for a diffusion process to approximate the random tour. This approximation is based on the fact that the variance of the diffusion process has the same functional form as $E[R^2_T]$, the mean squared radial displacement of a randomly touring target [Ref. 7]:

$$E[R_T^2] = 2V^2\lambda^2 \left(e^{-\frac{T}{\lambda}} - 1 + \frac{T}{\lambda} \right) \quad (16)$$

Furthermore, for the diffusion model, the target at all times has a circular normal distribution with mean $\mu=0$ and $\sigma^2 = \frac{1}{2}E[R_T^2]$. For a circular normal distribution, $R_T^2/2\sigma^2$ has a unit exponential distribution for which 86 percent of the density is contained within 2σ . [Ref. 5: p. 312]. For the random tour model, the normal assumption holds only for large T . Therefore the correction factor for $T/\lambda < 3.0$ is applied in equation (13).

It is possible, however, to derive an exact expression for $R86$. This is done by integrating the probability density function of the particle's position at time t , given by the formula:

$$g(x,y,t) = \frac{t}{2\pi\lambda(VT)^2\sqrt{1-\rho^2}} e^{-\frac{t}{\lambda}(1-\sqrt{1-\rho^2})} \quad \text{for } \rho < 1 \quad (17)$$

where $\rho \equiv (x^2+y^2)/(VT)^2$ [Ref. 8]. Setting up the integral and converting to polar coordinates, for $z < VT$ we have:

$$P(R_T \leq z) = \int_0^{2\pi} \int_0^z \frac{rT}{2\pi(VT)^2\lambda\sqrt{1-(r/VT)^2}} e^{-\frac{T}{\lambda}(1-\sqrt{1-(r/VT)^2})} dr d\theta \quad (18)$$

Integrating with respect to θ and letting $u=r/VT$, $du=dr/VT$, $\xi=z/VT$:

$$P(R_T \leq z) = \int_0^{\xi} \frac{u}{\sqrt{1-u^2}} \frac{T}{\lambda} e^{-\frac{T}{\lambda}(1-\sqrt{1-u^2})} du \quad (19)$$

Substituting the following:

$$\begin{aligned} v &= \sqrt{1-u^2} \\ dv &= -\frac{u}{\sqrt{1-u^2}} du \end{aligned} \quad (20)$$

we get:

$$\begin{aligned} P(R_T \leq z) &= \int_{\sqrt{1-\xi^2}}^1 \left(\frac{T}{\lambda} \right) e^{-\frac{T}{\lambda}(1-v)} dv \\ &= e^{-\frac{T}{\lambda}} \int_{\sqrt{1-\xi^2}}^1 \left(\frac{T}{\lambda} \right) e^{\frac{T}{\lambda}v} dv \\ &= e^{-\frac{T}{\lambda}} e^{\frac{T}{\lambda}v} \Big|_{\sqrt{1-\xi^2}}^1 \\ &= e^{-\frac{T}{\lambda}} \left[e^{\frac{T}{\lambda}} - e^{\frac{T}{\lambda}\sqrt{1-\xi^2}} \right] \\ &= 1 - e^{-\frac{T}{\lambda}(1-\sqrt{1-\xi^2})} \quad ; \xi < 1 \\ &= 1 \quad ; \xi \geq 1 \end{aligned} \quad (21)$$

The discontinuity at $\xi=1$ is due to the fact that with probability $e^{-T/\lambda}$ the target does not change course, so there is a probability of mass $e^{-T/\lambda}$ concentrated on the circle of radius VT.

We now solve for $R_{86}^2_{RT}$, the radius which contains 86 percent of the target density for the random tour case:

$$1 - e^{-\frac{T}{\lambda}(1-\sqrt{1-\xi^2})} = 0.86 \quad (22)$$

Solving for ξ :

$$\begin{aligned} \xi &= \sqrt{1 - \left(1 - \frac{2\lambda}{T}\right)^2} && ; T \geq 2\lambda \\ \xi &= 1 && ; T < 2\lambda \end{aligned} \quad (23)$$

where the integer 2 appears because $-\ln(1-0.86)=2$. Therefore:

$$\begin{aligned} R86_{RT}^2 &= V^2 T^2 \left[1 - \left(1 - \frac{2\lambda}{T}\right)^2 \right] \\ &= 4V^2 \lambda T - 4V^2 \lambda^2 && ; T \geq 2\lambda \\ R86_{RT}^2 &= V^2 T^2 && ; T < 2\lambda \end{aligned} \quad (24)$$

In Order to relate $R86$ and the number of diffusion iterations performed for the Area Clearance target, the developers of ASWTDA performed a series of experiments by applying the Area Clearance transition matrix repeatedly and fitting the data experimentally, obtaining the relation of Equation (14). Equation (14) can be derived analytically however. Furthermore, it can be shown that the values for a, b and c in q^{AC} can be varied so that the number of iterations required provides for more convenient application of the transition matrix.

First, we must show that the application of Markov target motion used in ASWTDA produces an approximately circular distribution. Assume we have a target at position $(0,0)$. Let X_n be the horizontal position at time n . Then:

$$X_n = \sum_{i=1}^n u_i \quad (25)$$

where u_i is the horizontal movement at time i . In ASWTDA, since the target may move at most one cell length, $u_i \in (-1, 0, 1)$ and, due to the symmetry of q^{AC} its expected value $E[u_i] = 0$. The variance, $\text{Var}[u_i] = E[u_i^2] - E[u_i]^2 = E[u_i^2]$. Using the values of the transition matrix, q^{AC} :

$$\begin{aligned} E[u_i^2] &= ((-1)^2 (2b+c) + (1)^2 (2b+c)) \\ &= (4b+2c) \end{aligned} \quad (26)$$

Since the u_i are independent and identically distributed random variables, by the Central Limit Theorem, X_n is approximately normally distributed with mean $\mu_x = E[X_n] = 0$ and variance $\sigma_x^2 = \text{Var}[X_n] = n(\text{Var}[u_i]) = n(4b+2c)$, for large n [Ref. 5: p. 289]. The same derivation can be applied to the vertical position

$$Y_n = \sum_{i=1}^n v_i \quad (27)$$

where v_i is the vertical movement at time i .

Therefore, (X_n, Y_n) is approximately bivariate normal with parameters $\mu = \mu_x = \mu_y = 0$, and $\sigma^2 = \sigma_x^2 = \sigma_y^2 = n(4b+2c)$. Furthermore, the covariance:

$$\begin{aligned} \text{Cov}[X_n, Y_n] &= E[X_n Y_n] - E[X_n]E[Y_n] \\ &= E[X_n Y_n] \end{aligned} \quad (28)$$

where the $(X_n, Y_n) \in (-n^2, \dots, -1, 0, 1, \dots, n^2)$ with probability distribution $[p(-n^2), \dots, p(-1), p(0), p(1), \dots, p(n^2)]$. Because of the symmetry of q^{AC} , we can see that:

$$\begin{aligned}
p(-n^2) &= P(X_n = -n, Y_n = n) + P(X_n = n, Y_n = -n) \\
&= P(X_n = n, Y_n = n) + P(X_n = -n, Y_n = -n) \\
&= p(n^2)
\end{aligned} \tag{29}$$

Thus:

$$\begin{aligned}
E[X_n Y_n] &= (n^2)p(n^2) + (-n^2)p(-n^2) + \dots + \\
&\quad (1)p(1) + (-1)p(-1) + (0)p(0) \\
&= 0
\end{aligned} \tag{30}$$

Therefore, $\text{Cov}[X_n, Y_n] = 0$ so X_n and Y_n are uncorrelated [Ref. 5: p.310]. The result is an approximately circular normal distribution with mean 0 and variance $n[4b+2c]$ when n is large. Note that this is true independent of the values of a , b and c . The important fact is that the transition matrix maintains horizontal and vertical symmetry. For most central limit theorem applications, large n is taken to be greater than 20. As an example, for a target speed of 15 knots, a mean time on leg of one hour and a cell size of 5 nautical miles, we find from Equations (13) and (14) that just over two hours would be required to meet this central limit theorem requirement.

We can now relate the number of iterations to the radius containing 86 percent of the target density, R_{86} . The radius $R = (X_n^2 + Y_n^2)^{1/2}$. Assuming the requirements of the central limit theorem are met, X_n and Y_n are Normally distributed, and because they are uncorrelated, they are also independent. Thus X_n/σ and Y_n/σ are independent standard normal random variables, and:

$$\frac{X_n^2}{\sigma^2} + \frac{Y_n^2}{\sigma^2} = \frac{X_n^2 + Y_n^2}{\sigma^2} \quad (31)$$

is approximately a Chi-Squared random variable with two degrees of freedom. As discussed earlier, 86 percent of the density is contained within two standard deviations, we will call this $R86_{TM}$, with the subscript TM representing the Transition Matrix method. Let W =(cell width), then for large n :

$$\begin{aligned} R86_{TM}^2 &= (2\sigma)^2 \\ &= n4(4b+2c)W^2 \\ &= n(16b+4c)W^2 \end{aligned} \quad (32)$$

where $R86_{TM}$ is measured in nautical miles. For the values of a , b and c used in q^{AC} , this results in the relationship:

$$R86^2 = 2.54nW^2 \quad (33)$$

This is very close to the experimentally obtained relationship of Equation (14) used in ASWTDA.

In ASWTDA, the Transition Matrix method is matched to the random tour by equating Equation (32) with $2E[R_T^2]$ from Equation (16), even when n is small. The method keeps a , b and c constant while making n be some integer other than the number of information updates (recall that these updates occur at regular intervals of $\Delta=6$ Min.). It would be more convenient to make n the same as the number of

information updates. If V is not too large compared to cell size, this can be achieved using the exact expression of Equation (24) instead of the approximate expression of Equation (13) by letting a , b and c depend on time. The crucial question is whether the increment $(16b+4c)W^2$ from Equation (32) can be made as large as the increase of Equation (24) in a period of length Δ . The slope of Equation (24) is at most $4V^2\lambda$ and the maximum value of $(16b+4c)$ is 4^1 , so the crucial inequality is $4V^2\lambda\Delta < 4W^2$. Letting $\Delta=0.1$ hour, we obtain the requirement:

$$V \leq W \sqrt{\frac{10}{\lambda}} \quad (34)$$

where V is measured in knots.

Note that for $T > 2\lambda$, the slope of Equation (24) is a constant value of $4V^2\lambda$, therefore, a , b and c may be constant for $T > 2\lambda$.

One can see from Equation (34) that there would be only few instances where a greater number of diffusions would be necessary. For example, for a mean time on leg of one hour and a cell size of five nautical miles, $V \leq 15.8$ Kts. These instances could be handled on a case by case basis, and n could still be made a convenient integer value.

¹ This occurs when $b=0.25$, i.e., all probability in the corner cells. Though it is recognized that this would result in some cells being inaccessible, it is the limiting case.

2. Primary Target Diffusion

In ASWTDA, the transition matrices are such that the target may move only as far as one cell with each application of q . This does not result in errors for the Area Clearance target, but the method of calculating the time interval, dt , for Primary Target diffusion, may result in some inaccuracy. For example, the Primary Target may be going faster than the average speed, and therefore may travel farther than one cell length in a time step dt , as illustrated in time step number 8 in Figure 4. The transition for this time step can either be ignored, or, as is done in ASWTDA, a count can be placed in the closest adjacent cell, inaccurate in either case. Thus there is a tradeoff involved in the choice of dt . Too large a dt results in the target being able to go farther than one cell length, resulting in the situation just described; too small a dt results in too few transitions to adjacent cells resulting in a high probability of remaining in the center cell of the transition matrix thus requiring more frequent application of the transition matrix, increasing computation time. In order to prevent a target from traveling farther than one cell length, the appropriate time step would be:

$$dt = \frac{\text{Cell Length}}{\text{Maximum Target Speed}} \quad (35)$$

It is not intended here to state an optimum method of choosing dt , simply that diffusion accuracy is dependent on this choice, which in turn depends on the target velocity distribution, and, as shown by the need for the cosine correction term in certain cases, on average target course as well. Note however that the average

target course has no meaning when the variance in target course is large, with the extreme case being a randomly touring target. This aspect of the problem warrants further study. Of course, another way to prevent this error is to expand the transition matrices, q , to say a 5×5 matrix. The tradeoff here would be an increase in both computation time and storage requirements.

VI. TARGET COMBINATION AND CREDIBILITY

A. TARGET COMBINATION

The question arises as to what specifically is described by the Area Clearance and Primary Target density matrices and the operations performed. Two elementary models are presented here, these are: 1. A single target which may be of one of two types, 2. Two separate targets. An attempt will be made in Section 3 to match each of these models with the operations performed in ASWTDA. This is not intended to represent all possible target models, merely two that were chosen, that have a sound probabilistic basis, to attempt to understand the operations of ASWTDA.

1. Single Target

In this scenario, it is known that there is a single target within the analysis area at time $t=0$, but it is not known of what type. Assume the target is of one of two types, **a** or **b**, and we presumably have some knowledge of the a priori probability distribution of target type. Furthermore, it is known that the type **a** target remains within the analysis area, and the type **b** target may leave the area and not reenter. For this scenario:

$$\begin{aligned}
A_{i,j}(t) &= P(E_{i,j}(t) \text{ and } a | H_t) \\
A_{i,j}^*(t) &= P(E_{i,j}(t) \text{ and } a | H_t I(t)) \\
B_{i,j}(t) &= P(E_{i,j}(t) \text{ and } b | H_t) \\
B_{i,j}^*(t) &= P(E_{i,j}(t) \text{ and } b | H_t I(t)) \\
C(t) &= P((\text{outside } S) \text{ and } b | H_t) \\
C^*(t) &= P((\text{outside } S) \text{ and } b | H_t I(t))
\end{aligned} \tag{36}$$

so $|A(t)| = P(a | H_t)$, likewise for target type b , and $|A(t) + B(t)| + C(t) = 1$.

There are two separate motion models involved described by the following transition matrices:

$$\begin{aligned}
Q^a(i,j|m,n) &= P(E_{i,j}(t+1) \text{ and } a | E_{m,n}(t) \text{ and } a) \\
Q^b(i,j|m,n) &= P(E_{i,j}(t+1) \text{ and } b | E_{m,n}(t) \text{ and } b)
\end{aligned} \tag{37}$$

Thus:

$$\begin{aligned}
A(t+1) &= A^*(t) \star Q^a \\
B(t+1) &= B^*(t) \star Q^b
\end{aligned} \tag{38}$$

and $C(t+1) = 1 - |A(t+1) + B(t+1)|$.

These matrices may be updated for received information using Bayes' theorem as follows:

$$\begin{aligned}
P(E_{i,j}(t) a | H_t I(t)) &= \frac{P(I(t) | E_{i,j}(t) a H_t) P(E_{i,j}(t) a | H_t)}{P(I(t) | H_t)} \\
P(E_{i,j}(t) b | H_t I(t)) &= \frac{P(I(t) | E_{i,j}(t) b H_t) P(E_{i,j}(t) b | H_t)}{P(I(t) | H_t)} \\
P((\text{outside } S) b | H_t I(t)) &= \frac{P(I(t) | (\text{outside } S) b H_t) P((\text{outside } S) b | H_t)}{P(I(t) | H_t)}
\end{aligned} \tag{39}$$

As discussed in Section II.C.1, we would expect to assume that $P(I(t) | E_{ij}(t)H_t a)$ and $P(I(t) | E_{ij}(t)H_t b)$ are both independent of H_t , but the nondetection probability may still, in principle, depend on target type.

To simplify notation, the following are defined:

$$\begin{aligned} a_{ij}(t) &= P(I(t) | E_{ij}(t) a H_t) \\ b_{ij}(t) &= P(I(t) | E_{ij}(t) b H_t) \\ c(t) &= P(I(t) | (\text{outside } S) b H_t) \end{aligned} \quad (40)$$

Thus, Equations (39) become:

$$\begin{aligned} A_{ij}^+(t) &= \frac{a_{ij}(t) A_{ij}(t)}{P(I(t) | H_t)} \\ B_{ij}^+(t) &= \frac{b_{ij}(t) B_{ij}(t)}{P(I(t) | H_t)} \\ C^+(t) &= \frac{c(t) C(t)}{P(I(t) | H_t)} \end{aligned} \quad (41)$$

and since:

$$P(I(t) | H_t) = \sum_{ij \in S} [a_{ij}(t) A_{ij}(t) + b_{ij}(t) B_{ij}(t)] + c(t) C(t) \quad (42)$$

we can see that $|A^+(t) + B^+(t)| + C^+(t) = 1$.

2. Two Separate Targets

In this model, there are two targets, one, a type **a** target is in the analysis area, and the other, type **b**, possibly outside the analysis area. For this model, the following are defined:

$$\begin{aligned}
A_{i,j}(t) &= P(E_{i,j}(t) | a H_t) \\
A_{i,j}^+(t) &= P(E_{i,j}(t) | a H_t, I(t)) \\
B_{i,j}(t) &= P(E_{i,j}(t) | b H_t) \\
B_{i,j}^+(t) &= P(E_{i,j}(t) | b H_t, I(t)) \\
C(t) &= P(\text{outside } S | b H_t) \\
C^+(t) &= P(\text{outside } S | b H_t, I(t))
\end{aligned} \tag{43}$$

Motion updating is equivalent to that for the single target model.

The density matrices are updated for received information using Bayes' theorem as follows:

$$\begin{aligned}
A_{i,j}^+(t) &= \frac{P(I(t) | E_{i,j}(t) a H_t) A_{i,j}(t)}{\sum_{i,j \in S} P(I(t) | E_{i,j}(t) a H_t) A_{i,j}(t)} \\
B_{i,j}^+(t) &= \frac{P(I(t) | E_{i,j}(t) b H_t) B_{i,j}(t)}{\sum_{i,j \in S} [P(I(t) | E_{i,j}(t) b H_t) B_{i,j}(t)] + P(I(t) | (\text{outside } S) b H_t) C(t)} \\
C^+(t) &= 1 - |B^+(t)|
\end{aligned} \tag{44}$$

3. Analysis

a. Single Target Model

To compare the operations performed in ASWTDA with the single target model as described above, we must make an assumption about the meaning of the target density matrices. Let T_{AC} and T_{PT} be the events the target is of type Area Clearance and Primary Target, respectively. We will assume the following:

$$\begin{aligned} AC_{ij}(t) &= P(E_{ij}(t) T_{AC} | H_t) \\ PT_{ij}(t) &= P(E_{ij}(t) T_{PT} | H_t) \end{aligned} \quad (45)$$

Note however from Equations (9) and (11) that following an information update, $|AC^+(t)| = |PT^+(t)| = 1$. ASWTDA then performs an additional operation to form what we will call $AC^{++}(t)$ and $PT^{++}(t)$, which is a renormalization of the density matrices such that:

$$\begin{aligned} AC^{++}(t) &= AC^+(t) | AC(t) | \\ PT^{++}(t) &= PT^+(t) | PT(t) | \end{aligned} \quad (46)$$

In the notation of the previous section, this is equivalent to:

$$\begin{aligned} A_{ij}^*(t) &= \frac{a_{ij}(t) A_{ij}(t)}{|a(t) A(t)|} |A(t)| \\ B_{ij}^*(t) &= \frac{b_{ij}(t) B_{ij}(t)}{|b(t) B(t)|} |B(t)| \end{aligned} \quad (47)$$

These equations must be compared with the results of Equations (41).

Using the very simplified, two cell example shown in Figure 5, these are clearly not equivalent calculations. Furthermore, although Reference 1 states that the Primary Target may leave the analysis area, following the target motion update, the Area Clearance matrix is again renormalized such that $|AC| = 1 - |PT|$. This is a deviation from the single target model in that, following each motion update, $C(t)=0$. So, rather than leave the area, the target type shifts to an Area Clearance target.

ASWTDA also handles positive information differently than negative information. As discussed in Chapter IV, $PT(t)=0$ for $t < t_0$. Thus, the Primary

Target distribution is based on Area Clearance type motion for $t < T_0$, and Primary target motion for $t \geq t_0$. This too is a deviation from the single target model and results in an inaccurate Primary Target distribution to a degree that is dependent upon the difference in the motion models and the elapsed time from $t=0$ to $t=t_0$.

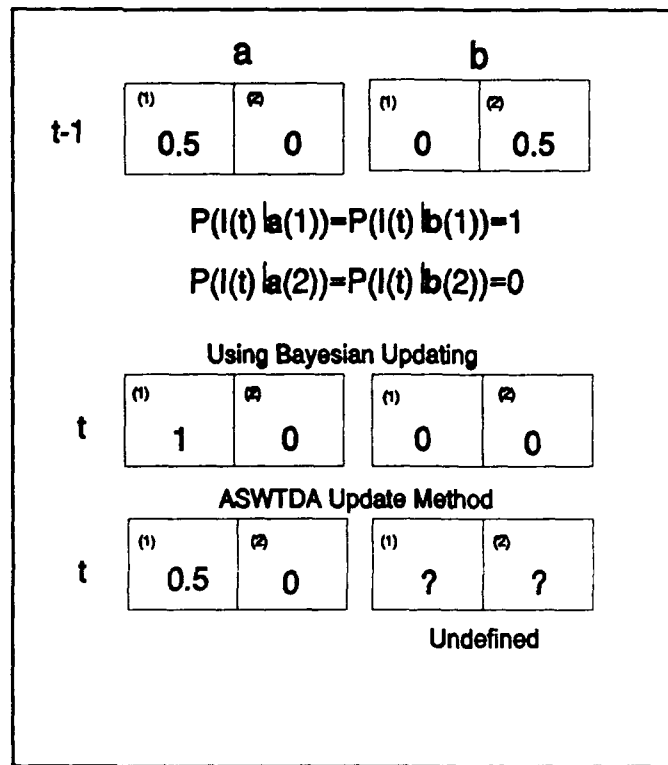


Figure 5 Comparison of Bayesian Update and ASWTDA Update Methods

As can be seen from Equations (39), both positive and negative information can be handled equally well using Bayes' theorem, provided the appropriate probabilities are known.

It is conceivable that receipt of positive information may alter the distribution of target type. This is properly handled by Equations (39). For example, if we make the assumption that some received positive information, $I(t)$, could not possibly be caused by target type **a**, in Equations (39), $P(I(t) | E_{ij}(t)aH_i) = 0$ for all (i,j) , thus $P(E_{ij}(t) | H_i I(t)) = 0$ for all (i,j) , and all probability is assigned to target type **b**. In order to perform this operation properly in ASWTDA, it is necessary that the density matrix for the Primary Target be nonzero for $t < t_0$, otherwise $P(I(t) | H_i) = 0$ which results in division by zero.

b. Two Target Model

To compare ASWTDA with the two target model, we must assume the target density matrices are defined as follows:

$$\begin{aligned} AC_{i,j}(t) &= P(E_{i,j}(t) | T_{AC} H_t) \\ AC_{i,j}^+(t) &= P(E_{i,j}(t) | T_{AC} H_t I(t)) \\ PT_{i,j}(t) &= P(E_{i,j}(t) | T_{PT} H_t) \\ PT_{i,j}^+(t) &= P(E_{i,j}(t) | T_{PT} H_t I(t)) \end{aligned} \tag{48}$$

The two target model clearly is not what is intended in ASWTDA, because, as stated in Reference 1, "...each of the density displays represents position information for the same single target [Ref. 1: p. 9]". This is further reinforced by the fact that, following each information and motion update, ASWTDA adjusts the density matrices such that $|AC| + |PT| = 1$.

4. Modifications to Comply With a One or Two Target Model

Changes may be made to ASWTDA so that the operations conform to either of the two motion models described above.

In order to conform to the single target model, the following changes must be made:

1. Maintain a Primary Target density matrix from time $t=0$, which follows the motion model that is believed to describe the Primary Target. If the motion model for the Primary Target is unknown, multiple possible scenarios could be initialized at $t=0$. Then, at $t=t_0$, the probability of each of these scenarios being the true motion model can be assessed. Another option is that upon receipt of positive information, a Primary Target motion model may be developed and started from $t=0$ to calculate a density $PT(t_0)$.

2. Initialize the Area Clearance and Primary Target prior distributions independently to what is believed, a priori, about the individual targets. If it is known that there is one target in the analysis area, then $|AC(0)| + |PT(0)| = 1$.
3. If the definitions given in the single target analysis section above are what is intended in ASWTDA, then updating for information should be performed using Equations (41).
4. As the Primary Target diffuses out of the analysis area, the Area Clearance matrix should not be renormalized to include this lost target density.
5. Since the operator is concerned about a single target, his attention should be focused on the combined target density, which is the best knowledge available on the single target.

If the intention in ASWTDA is that there are two targets, then in order to conform to the two target model, in addition to number 1 above, the following modifications must be made:

1. Initialize the Area Clearance and Primary Target prior distributions independently to what is believed, a priori, about the individual targets. Following the two target model, $|AC(0)| = 1$, and $|PT(0)| \leq 1$.
2. As the Primary Target diffuses out of the analysis area, only the Primary Target density matrix should be affected. Therefore $|PT(t+1)| \leq |PT^+(t)|$ whereas $|AC(t+1)| = |AC^+(t)| = 1$.
3. Update the target density matrices for information using Bayes' theorem of the form of Equations (44).
4. The density matrices should not be combined into a single target density matrix as they represent two separate targets.

B. CREDIBILITY

1. A Proper Application of Credibility

A decision aid must account for the fact that the emissions which resulted in the positive information report may have come from a target different from the target of interest. This may be as simple as assuming that it does not occur. Let T be the event the emissions are caused by the target of interest, let $I(t)$ be the event a positive report is made, and assume the following data are known:

$$\begin{aligned}\alpha_{ij}(t) &= P(I(t) T | E_{ij}(t)) \text{ for all } (i,j) \in S \\ \beta_{ij}(t) &= P(I(t) T^c | E_{ij}(t)) \text{ for all } (i,j) \in S \\ \gamma(t) &= P(I(t) T | \text{outside } S) \\ \eta(t) &= P(I(t) T^c | \text{outside } S)\end{aligned}\tag{49}$$

Thus $I(t)T^c$ is the event that a report is made and the target of interest is not the cause, and $\alpha_{ij}(t) + \beta_{ij}(t) = P(I(t) | E_{ij}(t))$, where $P(I(t) | E_{ij}(t))$ is discussed earlier in Section II.C.1. By the definition of conditional probability:

$$P(E_{ij}(t) | I(t)) = \frac{P(I(t) T | E_{ij}(t)) P(E_{ij}(t))}{P(I(t))} + \frac{P(I(t) T^c | E_{ij}(t)) P(E_{ij}(t))}{P(I(t))}\tag{50}$$

Note that $P(I(t))$ can be written as follows:

$$\begin{aligned}P(I(t)) &= \frac{P(I(t) T)}{P(T | I(t))} \\ &= \frac{P(I(t) T^c)}{P(T^c | I(t))}\end{aligned}\tag{51}$$

Letting $p_{ij}(t) = P(E_{ij}(t))$ and $P(t)$ be the matrix with elements $p_{ij}(t)$, and letting $\alpha(t)$ be the matrix with elements $\alpha_{ij}(t)$ and $\beta(t)$ the matrix with elements $\beta_{ij}(t)$, Equation (50) can be written:

$$\begin{aligned}
P(E_{ij}(t) | I(t)) &= \frac{P(I(t)T | E_{ij}(t))P(E_{ij}(t))}{P(I(t)T)} P(T | I(t)) + \frac{P(I(t)T^c | E_{ij}(t))P(E_{ij}(t))}{P(I(t)T^c)} P(T^c | I(t)) \\
&= \left(\frac{\alpha_{ij}(t)p_{ij}(t)}{|\alpha(t)P(t)| + \gamma(t)(1-|P(t)|)} \right) P(T | I(t)) + \left(\frac{\beta_{ij}(t)p_{ij}(t)}{|\beta(t)P(t)| + \eta(t)(1-|P(t)|)} \right) P(T^c | I(t))
\end{aligned} \quad (52)$$

Let $P(T | I(t))$ be an input called credibility, C . If we assume $\beta_{ij}(t) = \beta$ for all (i,j) and all t , i.e., if $I(t)$ is not caused by the target of interest, it does not matter where it is located, and if we make the further assumption that $\gamma(t) = \eta(t) = 0$ for all (t) , i.e., we can not possibly receive a contact report if the target is outside S , even if the target is not the cause of the report, Equation (52) becomes [Ref. 6: pp. 8-9]:

$$P(E_{ij}(t) | I(t)) = \left(\frac{\alpha_{ij}(t)p_{ij}(t)}{|\alpha(t)P(t)|} \right) C + p_{ij}(t)(1-C) \quad (53)$$

In other words, the posterior probabilities are an average of the prior probabilities (with probability $(1-C)$), and the posterior probabilities that would hold when $C=1$ (with probability C).

2. Credibility in ASWTDA

It is difficult to determine what is implied by the use of the confidence factor, cf , in ASWTDA. This is in part due to the fact that it is not clearly defined what is described by the Area Clearance and Primary Target density matrices. An attempt will be made to fit the above model, however, based on the single target model.

Let T be the event the emissions which resulted in the positive report were caused by the Primary Target, and $I(t_0)$ the event that positive information is received at time (t_0) . One infers from Reference 1 that the confidence factor, $cf = P(T | I(t_0))$,

i.e., cf is the probability that the received emissions were caused by the Primary Target given the receipt of the positive information at time (t_0) . For the analysis, the following assumptions will be made:

1. $B^D_{ij} = P(I(t_0)T | E_{ij})$ for all $(i,j) \in S$.
2. $C = P(T | I(t_0))$, i.e., C is the probability the emissions which resulted in the positive information report were caused by the Primary Target, given the receipt of positive information at time (t_0) .
3. $P(I(t_0)T^c | E_{ij}(t_0))$ is constant for all (i,j) .
4. $P(I(t_0)T | \text{outside } S) = P(I(t_0)T^c | \text{outside } S) = 0$

Equation (53) then becomes:

$$P(E_{ij}(t_0) \text{ and (Primary Target)} | I(t_0)) = P_{ij}^*(t_0) C + P_{ij}(t_0) (1 - C) \quad (54)$$

This is equivalent to Equation (10) with two exceptions:

1. The multiplication term, pr , used in ASWTDA is the confidence factor reduced further by excluding that portion of B^D which falls outside the analysis area or on land. Thus the confidence of the report is dependent upon the location of the analysis area in relation to the positive report.
2. The second term of Equation (54) is omitted altogether.

Thus we can see that, even given these assumptions about the target and the received information, the use of the confidence in ASWTDA falls short of a meaningful application of target credibility. This may be easily fixed by utilizing Equation (54) in ASWTDA, use of which would correspond to making assumptions 1, 2, 3 and 4 above.

VII. CONCLUSIONS AND RECOMMENDATIONS

In summary, there are four areas which need to be addressed in the continuing development of ASWTDA to prevent possibly erroneous results. These are:

1. The number of targets in the system
2. Proper incorporation of positive information
3. Primary target motion prior to receipt of positive information
4. The proper use of information credibility

The question of how many targets there are and what is meant by an Area Clearance target or a Primary Target has a profound affect on how the other points are handled. This issue must be resolved prior to addressing further issues. Once it is decided how many targets are being manipulated, it can be decided how the target or targets are to move, and to which target or targets positive information is applied. Also, it must be decided what is meant by the confidence factor in relation to positive information, and to apply it properly.

Other aspects which may not significantly degrade the results obtained from ASWTDA, but may affect the speed or simplicity of computation and overall solution accuracy are use of the exact expression for $R86$ for a randomly touring target and the adjusting of the values of a , b and c in the Area Clearance transition matrix so that the diffusion of the Area Clearance target occurs at regular convenient intervals.

LIST OF REFERENCES

1. Sonalysts, Inc., *AntiSubmarine Warfare Tactical Decision Aid (ASWTDA) Search Methodology*, Sonalysts, Inc., Nov. 1989.
2. *ASW Tactical Decision Aid Users Guide, Version 1.02*, Naval Sea Systems Command, Code 63D.
3. Telephone Conversation between LCDR Eric Rosenlof, OP-71, Chief of Naval Operations, Antisubmarine Warfare Division and the Author, 29 Jan. 1990.
4. Ross, S. M., *Introduction To Probability Models*, Academic Press, Inc., 1985.
5. Larson, H. J., *Introduction To Probability And Statistical Inference*, John Wiley and Sons, Inc., 1982.
6. Washburn, A., *Multiple Experts And Credibility In Search TDA's*, 1989.
7. Belkin, B., *The Ornstein-Uhlenbeck Displacement Process as a Model for Target Motion*, Daniel H. Wagner, Associates Interim Memorandum to Applied Physics Laboratory, February 1, 1978.
8. Washburn, Alan, "Probability Density of a Moving Particle", *Operations Research*, Vol. 17, No. 5, September-October, 1969.

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